Rossmoyne Senior High School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS METHODS YEAR 12 (ATMAM)

Section One: Calculator-free

Circle your Teacher's Name: Alvaro Koulianos | Bestall Luzuk | Fraser-Jones Murray | Kigodi Tanday

Student number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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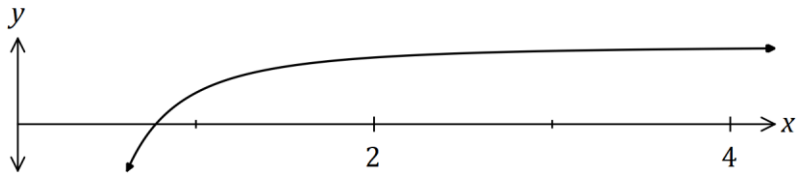
Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

The curve shown below passes through the point $(1, 4)$ and is such that $\frac{dy}{dx} = \frac{12}{x^3}$.



(a) Determine the equation of the curve. (3 marks)

Solution
$y' = 12x^{-3} \Rightarrow y = -6x^{-2} + c$ $4 = -6 + c \Rightarrow c = 10$ $y = 10 - \frac{6}{x^2}$
Specific behaviours
✓ integrates derivative ✓ determines constant ✓ states equation

(b) Determine the area of the region enclosed by the curve, the x-axis, the line $x = 1$ and the line $x = 3$. (3 marks)

Solution
$A = \int_1^3 10 - 6x^{-2}\, dx$ $= \left[10x + \frac{6}{x}\right]_1^3$ $= 32 - 16 = 16 \text{ sq units}$
Specific behaviours
✓ writes integral with bounds ✓ integrates ✓ evaluates integral **NB:** Award 1 mark if students integrate derivative function instead of curve (will give $\frac{16}{3}$)

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Question 2 **(7 marks)**

(a) Determine

(i) $\dfrac{d}{dx}\left(\dfrac{e^{5x+3}}{\cos(2x+\pi)}\right)$. (3 marks)

Solution
$\dfrac{(5e^{5x+3})(\cos(2x+\pi)) - (e^{5x+3})(-2\sin(2x+\pi))}{\cos^2(2x+\pi)}$
Specific behaviours
✓ correctly applies quotient rule ✓ derivative of numerator ✓ derivative of denominator

(ii) $\dfrac{d}{dt}\displaystyle\int_t^2 (3x-1)^2\ dx$. (2 marks)

Solution
$-\dfrac{d}{dt}\displaystyle\int_2^t (3x-1)^2\,dx = -(3t-1)^2$
Specific behaviours
✓ swap limits and negate expression ✓ simplifies, using correct variable

(b) Simplify the indefinite integral $\displaystyle\int (4x-1)^2\ dx$. (2 marks)

Solution
$\dfrac{(4x-1)^3}{3\times 4} + c = \dfrac{(4x-1)^3}{12} + c$
Specific behaviours
✓ antidifferentiates ✓ simplifies and includes constant **NB:** Full marks if binomial is expanded first and integration is correct, no marks just for expanding if integration is incorrect.

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Question 3 **(7 marks)**

A calculator program will generate a single random integer n, where $2 \leq n \leq 11$. The program is run once, and the random variable X is the number of integers less than 10 obtained.

(a) Explain why X is a Bernoulli random variable. (1 mark)

Solution
In a single trial, X will be 1 or 0 - either an integer less than 10 is generated ($X = 1$) or not ($X = 0$).
Specific behaviours
✓ explains event will or will not happen

(b) Determine $P(X = 1)$. (1 mark)

Solution
$P(X = 1) = \frac{8}{10} = \frac{4}{5}$
Specific behaviours
✓ correct probability

(c) Determine the mean and standard deviation of X. (2 marks)

Solution
$\bar{X} = \frac{4}{5}$, $\sigma_X = \sqrt{\frac{4}{5} \times \frac{1}{5}} = \frac{2}{5}$
Specific behaviours
✓ mean ✓ standard deviation

The random variable Y is the number of integers less than 10 obtained in three consecutive runs of the program.

(d) Determine $P(Y \leq 1)$. (3 marks)

Solution
$P(Y = 0) = \left(\frac{1}{5}\right)^3 = \frac{1}{125} = 0.008$ $P(Y = 1) = \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right) \times 3 = \frac{12}{125} = 0.096$ $P(Y \leq 1) = \frac{13}{125} = 0.104$
Specific behaviours
✓ $P(Y = 0)$ ✓ $P(Y = 1)$ ✓ correct probability Note: accept decimals or fractions

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Question 4 **(5 marks)**

Let $f(x) = 2x + \frac{k}{3x}$, $x > 0$ and k is a constant. The graph of $y = f(x)$ has a stationary point when $x = 4$.

(a) Determine the value of k. (2 marks)

Solution
$f'(x) = 2 - \frac{k}{3x^2}$ $f'(4) = 0 \Rightarrow 2 = \frac{k}{48} \Rightarrow k = 96$
Specific behaviours
✓ $f'(x)$ ✓ value of k

(b) Use the second derivative test to determine the nature of the stationary point. (3 marks)

Solution
$f''(x) = \frac{d}{dx}\left(2 - \frac{32}{x^2}\right) = \frac{64}{x^3}$ $f''(4) = \frac{64}{64} = 1$ Hence stationary point is a minimum because $f''(4) > 0$
Specific behaviours
✓ $f''(x)$ ✓ evaluates sign of $f''(4)$ ✓ correct nature of point

Question 5 **(7 marks)**

A farmer keeps a brood of n hens that can each lay up to one egg per day. On any given day, the probability that a hen lays an egg is independent with a constant value of p.

The discrete random variable X is the number of eggs laid by the brood in one day and X has a mean of 5 and standard deviation of 2.

(a) State the name given to this type of probability distribution and briefly explain why it is discrete. (2 marks)

Solution
Binomial. Discrete as X can only be one of a specified set of values.
Specific behaviours
✓ name ✓ explanation

(b) Determine the value of n and the value of p. (3 marks)

Solution
$np = 5$ and $np(1-p) = 2^2$ $1 - p = \frac{4}{5} = 0.8 \Rightarrow p = \frac{1}{5} = 0.2$ $\frac{n}{5} = 5 \Rightarrow n = 25$
Specific behaviours
✓ writes simultaneous equations ✓ value of p ✓ value of n

(c) Determine the mean and variance of the distribution Y, where $Y = 5X + 3$. (2 marks)

Solution
$\bar{Y} = 5 \times 5 + 3 = 28$ $\sigma_Y^2 = (5 \times 2)^2 = 100$
Specific behaviours
✓ value of $\bar{Y}$ ✓ value of σ_Y^2

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Question 6 **(6 marks)**

(a) Differentiate the following (solutions should have positive indices where appropriate):

(i) $y = \left(\frac{1}{e^{3x}}\right)^4$ (2 marks)

Solution
$y = (e^{-3x})^4$ $= e^{-12x}$ $\frac{dy}{dx} = -12e^{-12x}$ $= -\frac{12}{e^{12x}}$
Specific behaviours
✓ differentiates ✓ writes solution with positive indices

(ii) $y = 2e^{\cos x}$ (2 marks)

Solution
$\frac{dy}{dx} = -2\sin x.e^{\cos x}$
Specific behaviours
✓ correct derivative of $\cos x$ ✓ solution includes $2e^{\cos x}$

(b) Use calculus rules to find the gradient function of the following (do not simplify):

$f(x) = \frac{\cos x}{1-e^{2x}}$ (2 marks)

Solution
$f'(x) = \frac{-\sin x(1-e^{2x}) - \cos x(-2e^{2x})}{(1-e^{2x})^2}$
Specific behaviours
✓ applies quotient rule or product rule with negative indices for denominator ✓ correct solution

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Question 7 **(7 marks)**

A curve has equation $y = 5xe^{2ax}$, where a is a positive constant.

(a) Determine, in terms of a, the coordinates of the stationary point of the curve. (4 marks)

Solution
$\frac{dy}{dx} = 5e^{2ax} + 10axe^{2ax}$ $5e^{2ax}(1 + 2a) = 0$ $x = -\frac{1}{2a}$ $y = -\frac{5e^{-1}}{2a}$ $\left(\frac{-1}{2a}, \frac{-5}{2ae}\right)$
Specific behaviours
✓ applies product rule ✓ equates derivative to zero ✓ solves for x-coordinate ✓ correct coordinates

(b) Determine the coordinates of the point of inflection of the curve when $a = \frac{1}{10}$. (3 marks)

Solution
$\frac{dy}{dx} = 5e^{2\left(\frac{1}{10}\right)x} + 10\left(\frac{1}{10}\right)xe^{2\left(\frac{1}{10}\right)x}$ $= 5e^{\frac{x}{5}} + xe^{\frac{x}{5}}$ $\frac{d^2y}{dx^2} = e^{\frac{x}{5}} + e^{\frac{x}{5}} + \frac{x}{5}e^{\frac{x}{5}}$ $e^{\frac{x}{5}}\left(2 + \frac{x}{5}\right) = 0 \Rightarrow x = -10$ $y = -50e^{2\left(\frac{1}{10}\right)(-10)} = -50e^{-2}$ $\left(-10, \frac{-50}{e^2}\right)$
Specific behaviours
✓ correct second derivative ✓ solves for x-coordinate ✓ correct coordinates

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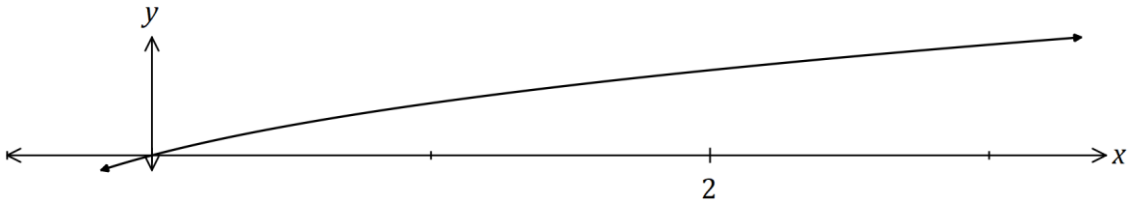
Question 8 **(7 marks)**

(a) Determine $\frac{d}{dx}\left(4x\sqrt{x+1}\right)$. (2 marks)

Solution
$\frac{d}{dx}(4x\sqrt{x+1}) = 4\sqrt{x+1} + \frac{2x}{\sqrt{x+1}}$
Specific behaviours
✓ applies product rule ✓ applies chain rule

(b) Part of the graph of $y = \frac{2x}{\sqrt{x+1}}$ is shown below.



Using your answer from part (a), determine $\int_0^3 \frac{2x}{\sqrt{x+1}}\,dx$. (5 marks)

Solution
$\int \frac{d}{dx}(4x\sqrt{x+1})\,dx = \int 4\sqrt{x+1}\,dx + \int \frac{2x}{\sqrt{x+1}}\,dx$ $\int \frac{2x}{\sqrt{x+1}}\,dx = 4x\sqrt{x+1} - \int 4(x+1)^{\frac{1}{2}}\,dx$ $= 4x\sqrt{x+1} - \frac{8}{3}(x+1)^{\frac{3}{2}} + c$ $\int_0^3 \frac{2x}{\sqrt{x+1}}\,dx = \left[4x\sqrt{x+1} - \frac{8}{3}(x+1)^{\frac{3}{2}}\right]_0^3$ $= \left(12\sqrt{4} - \frac{8}{3}(4)^{\frac{3}{2}}\right) - \left(0 - \frac{8}{3}(1)^{\frac{3}{2}}\right)$ $= \left(24 - \frac{64}{3}\right) + \frac{8}{3}$ $= \frac{16}{3}$
Specific behaviours
✓ equation using integrals from answer (a) ✓ uses $\int f'(x)\,dx = f(x)$ ✓ integrates $4\sqrt{x+1}$ ✓ substitutes bounds ✓ correct area

End of questions

Supplementary page

Question number: _____